By introducing more and more precise examples, we take on little by little the generalization of the Q/D GreenLab equation.

5. Branching + Biomass + fruits only on branches

When trunk and branches do not bear the same number of organs[3], equation (1) is written as follows

$$\boxed{Q(n) = \frac{E}{re} \frac{N_{\varphi 1}(n)\, n_{a1} p_{a1} + N_{\varphi 2}(n)\, n_{a2} p_{a2}}{D(n)} Q(n-1)} \qquad (8)$$

Demonstration

$D_{\varphi 1}(n) = N_{\varphi 1}(n) \left(n_{a1} p_{a1} + n_{e1} p_{e1}\right)$ for organs of physiological age 1

$D_{\varphi 2}(n) = N_{\varphi 2}(n) \left(n_{a2} p_{a2} + n_{e2} p_{e2} + n_{f2} p_{f2}\right)$ for organs of physiological age 2 (with fruits)

$D_{\varphi 1}(n)$ = total demand of trunk phytomeres at n cycle

$D_{\varphi 2}(n)$ = total demand of branches phytomeres at n cycle

$N_{\varphi 1}(n)$ = total number of trunk phytomeres at n cycle

$N_{\varphi 2}(n)$ = total number of branches phytomers at n cycle

The total demand is :

$$D(n) = D_{\varphi 1}(n) + D_{\varphi 2}(n) = N_{\varphi 1}(n) \left(n_{a1} p_{a1} + n_{e1} p_{e1}\right) + N_{\varphi 2}(n) \left(n_{a2} p_{a2} + n_{e2} p_{e2} + n_{f2} p_{f2}\right)$$

Let

$$d_{\varphi 1} = n_{a1} p_{a1} + n_{e1} p_{e1}$$
$$d_{\varphi 2} = n_{a2} p_{a2} + n_{e2} p_{e2} + n_{f2} p_{f2}$$

$d_{\varphi 1}$ = demand of one trunk phytomere

$d_{\varphi 2}$ = demand of one branch phytomere

Then

$$D(n) = N_{\varphi 1}(n)\, d_{\varphi 1} + N_{\varphi 2}(n)\, d_{\varphi 2}$$

but

$N_{\varphi 1}(n) = 1$; only one active phytomer on the trunk at cycle n (the last internode at trunk top)

$N_{\varphi 2}(n) = n-1$; there are $n-1$ active phytomers on the branches at cycle n (corresponding to the apical internode of branches)

[3] In the following equations, phytomers numbered 1 are those of the trunk (physiological age 1), and the phytomers numbered 2 are those of the branches (physiological age 2)

We obtain $D(n) = d_{\varphi 1} + (n-1)\, d_{\varphi 2}$

If we take equation (1) that links total demand at n cycle and total supply at $n-1$ cycle

$$Q(n) = \frac{E}{re} \frac{N_{\varphi 1}(n)\, n_{a1}\, p_{a1} + N_{\varphi 2}(n)\, n_{a2}\, p_{a2}}{D(n)} Q(n-1) = \frac{E}{re} n_{a1}\, p_{a1} \frac{Q(n-1)}{D(n)} + (n-1) \frac{E}{re} n_{a2}\, p_{a2} \frac{Q(n-1)}{D(n)}$$

Let $k_1 = \frac{E}{re} n_{a1}\, p_{a1}$ and $k_2 = \frac{E}{re} n_{a2}\, p_{a2}$, we have then

$$Q(n) = \left(\frac{k_1}{d_{\varphi 1} + (n-1)\, d_{\varphi 2}} + \frac{(n-1)\, k_2}{d_{\varphi 1} + (n-1)\, d_{\varphi 2}} \right) Q(n-1)$$

When $n \to \infty$ the first term tends to 0 and the second tends to $\frac{k_2}{d_{\varphi 2}}$. Then $Q(n) \to \frac{k_2}{d_{\varphi 2}} Q(n-1)$.

$$\lim_{n \to \infty} Q(n) = \frac{E}{re} \frac{n_{a2}\, p_{a2}}{d_{\varphi 2}} Q(n-1)$$

If $\frac{k_2}{d_{\varphi 2}} = 1$ then we have progressively $Q(n) \to Q(n-1)$

And if $E = 1, n_{a2} = 1, p_{a2} = 1$ then $re = 1$ when $n \to \infty$

Numerical application

For this example model, trunk phytomers bear one leaf, the first internode of branches and the branches phytomers bear one leaf and four fruits. ***What are the sink strengths of trunk internodes, branch internodes and fruits (of branches) at equilibrium?*** ($E = 1$, $r = 10$, $e = 0.05$)

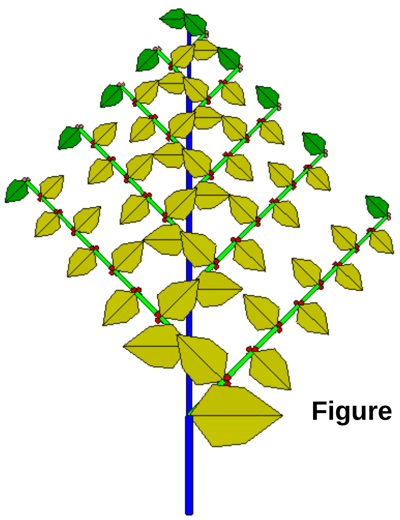

Figure 8. Roux's model at equilibrium with fruits on branches