Computational modeling for forest dynamics with Markov model individual-centered

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Abstract— **The forests are in fact ecological systems of great complexity which present *interaction phenomena* associated with the *competition* between individuals of the same species but also between individuals of different species. This competition is about access to resources (light, water, nutrients,...). The *scales* of forest ecosystems are very long. For this reason we wish to use in this work Markov modeling of the spatial distribution of individuals in the form of an individual-based model through the stochastic branching process.**
Keywords— **Forests dynamics, Branching stochastic process, interaction, competition, individual-based model (IBM).**

I. INTRODUCTION

This work does not pretend to offer a faithful model of forest ecological systems, but to propose a individual-centered model taking into account two key features:

(i) Birth and Dispersal: each individual is capable of give birth to a new tree located in the vicinity of the parent tree (Dispersal).

(ii) Death: the death is due to natural causes or to competition for access to natural resources.

The growth will not be discussed in this work.

Markov models of particles interacting in continuous time and space, branching (birth/death), are particularly adapted to this situation. We are particularly interested to a model developed by Méléard–Fournier [1]. This model had originally been proposed by Bolker, Pacala [2]. The treatments in these two items were different.

Bolker, Pacala [2] describe the Bolker equation, called the Kolmogorov equation, which describes the evolution of the law of the ecosystem process. This equation is complex and "live" in a large space. The authors then propose an approximation method of moments approximation allows to find the first two moments of the exact solution.

Méléard–Fournier [1] used this model and provides a rigorous mathematical formulation. They also describe the Markov process underlying with a form of Monte Carlo algorithms.

We present the model in the simple form and we studied the numerical simulation tests. The numerical simulations were made in `MatLab`.

II. A MODEL FOR FOREST DYNAMICS

II.1. Modeling forest dynamics

We Consider the forest:

$$\mathcal{X} = [x^1_{min}, x^1_{max}] \times [x^2_{min}, x^2_{max}] \subset \mathbb{R}^2$$

for simplicity, we use one hectare of the forest (100 square meters of hand). We would propose a simple model of forest dynamics: it will consist of a single species and we only consider the location of trees (individuals). For every time t, the forest will be represented by

$$\nu_t \stackrel{\text{def}}{=} \{x^i_t\,;\, i = 1 \cdots N_t\} \subset \mathcal{X}$$

Each individual i will be represented by its position x^i_t and $N_t = |\nu_t|$ (cardinal of ν_t) means the population size at time t.

The forest will be subject to ponctuels mechanisms. Starting from a forest ν and individual $x \in \nu$

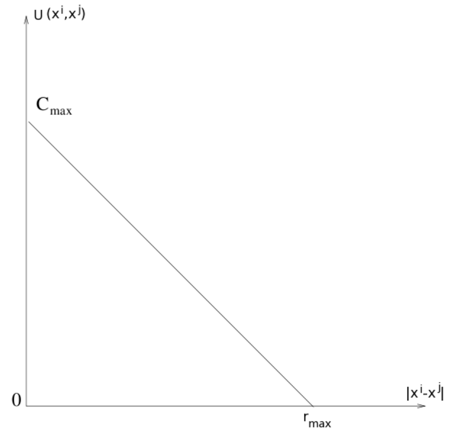


Fig. 1 : Figure illustrating the parameters of the function of competition.

(i) The individual x can die a natural death with a rate λ^m and then:

$$\nu \mapsto \nu \setminus \{x\}$$

(the forest loses the individual x).

(ii) The individual x can give birth to a new individual with a rate λ^n, position x' of this new individual is determined by a dispersal kernel $D(dx)$ and then:

$$\nu \mapsto \nu \cup \{x'\}$$

(the forest earns new individual x').

(iii) The individual x can die due to competition modeled by a rate $\lambda^c(x,\nu)$ which depends on x and ν, then:

$$\nu \mapsto \nu \setminus \{x\}$$

(the forest loses the individual x).

Assume again that these basic mechanisms are independent. We can consider more complex models where the rate may depend on the position in space. We may also consider a growth model: in addition to his position, each individual is also characterized by a size. We here prefer to keep the simple model.

II.2. Dispersal kernel

For the dispersal kernel we consider a Gaussian kernel $D(dx') = \mathcal{N}(0, \sigma^2 I)$. To easily manage the boundary conditions we assume that $\mathcal{X}$ is a torus.

II.3. competition kernel

For competition rates, we consider:

$$\lambda^c(x,\nu) = \sum_{y\in\nu} u(x,y)$$

where

$$u(x,y) = \begin{cases} C_{\max}\left(1 - \frac{1}{r_{\max}}\,||x-y||\right)^+ & \text{if } x \neq y\,, \\ 0 & \text{else.} \end{cases}$$

This interaction kernel means that: more the individual x is surrounded by neighbors more it is subject of disappears (see Figure 1).

III. Numerical simulations

Let the last event T_{k-1} and the corresponding forest $\nu_{T_{k-1}}$, we simulate t_k and ν_{t_k} as follows: calculates "global clock" events given by

$$m_k = m_k^{\rm n} + m_k^{\rm m} + m_k^{\rm c}$$

with

$$m_k^{\mathrm{n}} = \lambda^n\,|\nu_{T_{k-1}}|\,, \qquad m_k^{\mathrm{m}} = \lambda^m\,|\nu_{T_{k-1}}|\,, \qquad m_k^{\mathrm{c}} = C_{\max}\,|\nu_{T_{k-1}}|^2.$$

we set $T_k = T_{k-1} + S_k$ with $S_k \sim \mathrm{Exp}(m_k)$, and $\nu_t = \nu_{T_{k-1}}$ for $t \in [T_{k-1}, T_k[$.
We calculate the probabilities:

$$\alpha_k^{\mathrm{n}} = m_k^{\mathrm{n}}/m_k\,, \qquad \alpha_k^{\mathrm{m}} = m_k^{\mathrm{m}}/m_k\,, \qquad \alpha_k^{\mathrm{n}} = m_k^{\mathrm{c}}/m_k$$

We draw the event as follows:

- with probability α_k^{n}: Birth. We draw the individual i in $\{1,\dots,|\nu_{T_{k-1}}|\}$. We draw $z \in \mathbb{R}^d$ with dispersal kernel $D(dz)$. It adds a new individual to the position $x' = x^i_{\nu_{T_{k-1}}} + z$ and we set:

$$\nu_{T_k} = \nu_{\nu_{T_{k-1}}} \cup \{x'\}\,.$$

- with probability α_k^{m}: natural death. We draw the individual i in $\{1,\dots,|\nu_{T_{k-1}}|\}$ and we set:

$$\nu_{T_k} = \nu_{\nu_{T_{k-1}}} \setminus \{x^i_{\nu_{T_{k-1}}}\}\,.$$

- with probability α_k^{c}: death due to competition. We draw two individuals i and j in $\{1,\dots,|\nu_{T_{k-1}}|\}$. It rejects the event of death due to competition with probability $1-(u(x^i_{\nu_{T_{k-1}}}, x^j_{\nu_{T_{k-1}}})/C_{\max})$, otherwise it removes the individual i and we set

$$\nu_{T_k} = \nu_{\nu_{T_{k-1}}} \setminus \{x^i_{\nu_{T_{k-1}}}\}\,.$$

III.1. Test 1: High dispersion

The forest of high dispersion of individuals in the forest $\bar{\mathcal{X}}$ has been simulated using these parameters:

Parameters:	λ^n	λ^m	$C_{\max}$	$r_{\max}$	σ
Values:	2	1	2.5 10^{-3}	100	$\sqrt{5}$

After their births, the dispersion of new trees is even stronger than the standard deviation of Gaussian kernel is high.

III.2. Test 2: Low dispersion

The forest shows clusters in the forest $\bar{\mathcal{X}}$ has been simulated using these parameters:

Parameters:	λ^n	λ^m	$C_{\max}$	$r_{\max}$	σ
Values:	2	1	2.5 10^{-3}	100	$\sqrt{0.01}$

So when the standard deviation of Gaussian kernel is small, the dynamics of the forest evolves through a process of clustering.

III.3. Test 3: High competition

The forest of low density was simulated using these parameters:

Parameters:	λ^n	λ^m	$C_{\max}$	$r_{\max}$	σ
Values:	2	1	3 10^{-3}	100	1

So, if competition is strong, we get a phenomenon of desertification of the forest, which prevents the growth of the tree density in $\bar{\mathcal{X}}$.

III.4. Test 4: Low competition

The forest of high density was simulated using these parameters:

Parameters:	λ^n	λ^m	$C_{\max}$	$r_{\max}$	σ
Values:	2	1	3 10^{-4}	100	1

So, if competition is low, while the forest invaded the field $\bar{\mathcal{X}}$ which leads to strong growth in tree density.

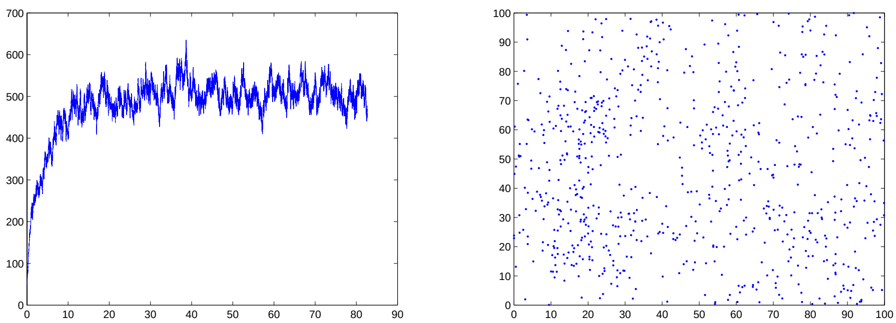
draw ν_0 and set $T_0 = 0$
for $k = 1, 2, \dots$ **do**
  $N \leftarrow |\nu_{T_{k-1}}|$

  % setting clocks
  $m^{\mathrm{n}} \leftarrow \lambda^n N$
  $m^{\mathrm{m}} \leftarrow \lambda^m N$
  $m^{\mathrm{c}} \leftarrow C_{\max} N^2$
  $m \leftarrow m^{\mathrm{n}} + m^{\mathrm{m}} + m^{\mathrm{c}}$

  % new instant event
  $S \sim \mathrm{Exp}(m)$
  $T_k \leftarrow T_{k-1} + S$

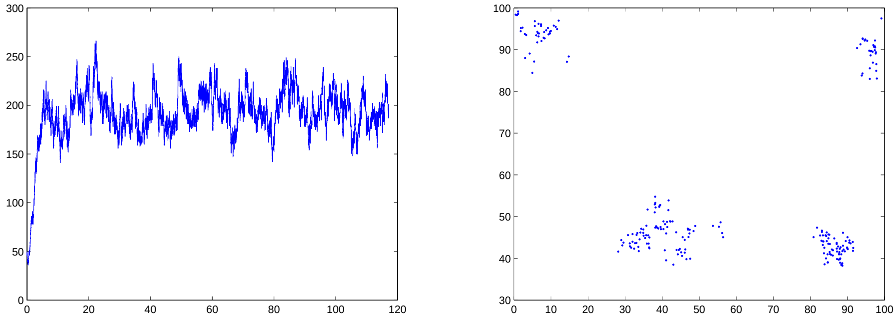
  $u \sim \mathcal{U}[0, 1]$
  $i \sim \mathcal{U}\{1, \dots, N\}$
  if $u \leq m^{\mathrm{n}}/m$ **then**
    $z \sim D(dz)$
    $\nu_{T_k} \leftarrow \nu_{T_{k-1}} \cup \{x^i_{\nu_{T_{k-1}}} + z\}$ % birth
  else if $u \leq (m^{\mathrm{n}} + m^{\mathrm{m}})/m$ **then**
    $\nu_{T_k} \leftarrow \nu_{T_{k-1}} \setminus \{x^i_{\nu_{T_{k-1}}}\}$ % naturel death
  else
    $j \sim \mathcal{U}\{1, \dots, N\}$
    $\beta \sim \mathcal{U}[0, 1]$
    if $\beta \leq u(x^i_{\nu_{T_{k-1}}}, x^j_{\nu_{T_{k-1}}})/C_{\max}$ **then**
      $\nu_{T_k} \leftarrow \nu_{T_{k-1}} \setminus \{x^i_{\nu_{T_{k-1}}}\}$ % death by competition
    end if
  end if
end for

ALG. 1: *Simulation algorithm of the forest.*



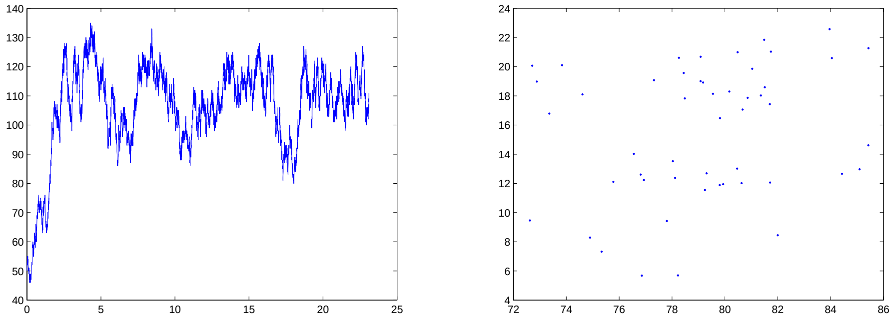
(a) *Dynamics of individuals number in time.* (b) *Final state of forest dynamics simulation.*

Fig. 2 : Test 1: High dispersion (dynamics of individuals number in time and spatial distribution of forest).



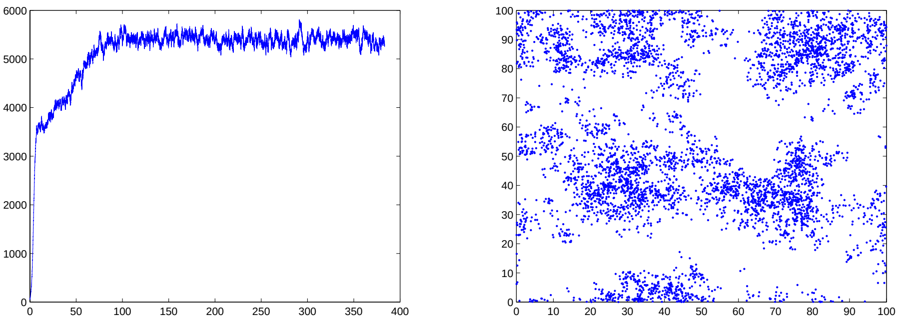
(a) *Dynamics of individuals number in time.* (b) *Final state of forest dynamics simulation.*

Fig. 3 : Test 1: Low dispersion (dynamics of individuals number in time and spatial distribution of forest).



(a) *Dynamics of individuals number in time.* (b) *Final state of forest dynamics simulation.*

Fig. 4 : Test 1: High competition (dynamics of individuals number in time and spatial distribution of forest).



(a) *Dynamics of individuals number in time.* (b) *Final state of forest dynamics simulation.*

Fig. 5 : Test 1: Low competition (dynamics of individuals number in time and spatial distribution of forest).

IV. Parameter estimation by stochastic gradient

Although we had data from the UMR "Dynamics of Natural Forests" of CIRAD we preferred–before addressing the statistical problems - check to what extent the kind of model proposed here is "controllable" or not.

To do this, we set a target characteristic such as population size N^*. The problem is whether it is possible to estimate a parameter, like the birth rate λ^n, and find empirical population average of N^*. The aim is obviously to do that automatically.

We use a stochastic gradient algorithm, particularly as it called Kiefer-Wolfowitz. The algorithms combine the stochastic gradient optimization method of the gradient and the Monte Carlo method to optimize a criterion written in the expectancy form.

Here we wish to minimize, for k large, the criterion:

$$J(\lambda^n) = \mathbb{E}_{\lambda^n}(|\nu_{T_k}| - N^*)^2.$$

ATTENTION: here $|\nu_{T_k}|$ is the cardinal of the set ν_{T_k}. $\mathbb{E}_{\lambda^n}$ means that the process is generated by the value of λ^n.

To be more rigorous, it must be understood for "k large" in a sense ergodic: the rigorous criterion would $\lim_{T\uparrow\infty} \frac{1}{T}\int_0^T (|\nu_t| - N^*)^2\,dt$.

The stochastic gradient algorithm is written:

$$\lambda^n_{k+1} \leftarrow \lambda^n_k - a_k\,\frac{\hat{J}(\lambda^n_k + c_k) - \hat{J}(\lambda^n_k - c_k)}{2\,c_k}, \tag{1}$$

with

(i) $\hat{J}(\lambda^n_k \pm c_k)$ is the empirical approximation $J(\lambda^n_k \pm c_k)$ calculated by simulating a forest setting $\lambda^n_k \pm c_k$ and using a moving average;

(ii) parameters a_k and c_k are chosen such that:

$$\sum_k a_k\,c_k < \infty \quad \text{and} \quad \sum_k a_k^2/c_k^2 < \infty\,.$$

In practice it is advisable to take $a_k = a \times k^{-1}$ and $c_k = c \times k^{-1/6}$ with a and c two positive constants [3].

(iii) In (1) it is necessary to project at each iteration the estimate in a given interval $[\lambda^n_{min}, \lambda^n_{max}]$.

The suites a_k and c_k depend repeatedly of k. The choice of the constants a and c is crucial for the speed of convergence of the algorithm. Indeed, if we use two small values of a and c at the start then, as shown in Figure **??**, we obtain a convergence with low speed. Similarly, if we use two values higher or lower of a and c, we obtain convergence with fast speed as seen in Figure **??** for the size of the forest and also for parameter in this example is the birth rate λ^n.

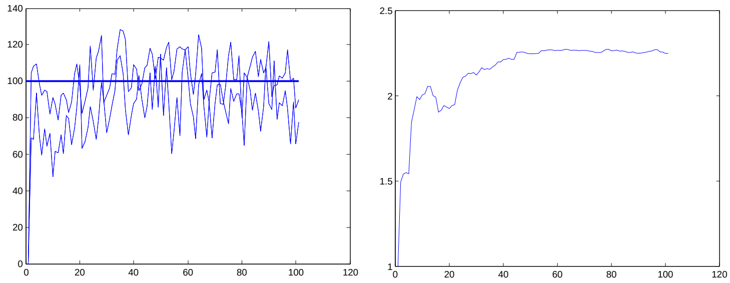
```
choose λ_0^n
for k = 1, 2, ... do
    a_k = a/k
    c_k = c/k^{1/6}

    % simulation of forest (see. Alg. 1):
    simulate T_k^± with the parameter λ_k^n ± c_k
    T_k ← T_k^+ ∧ T_k^-
    simulate two forests: ν_{T_k}^{λ_k^n ± c_k}

    % population sizes empirical:
    N̂^± ← (1/L) Σ_{ℓ=k-L}^{k} |ν_{T_ℓ}^{λ_ℓ^n ± c_ℓ}|

    % iteration of Kiefer-Wolfowitz:
    λ_{k+1}^n ← λ_k^n + a_k ((N̂^+ − N^*)^2 − (N̂^- − N^*)^2) / (2 c_k)
end for
```

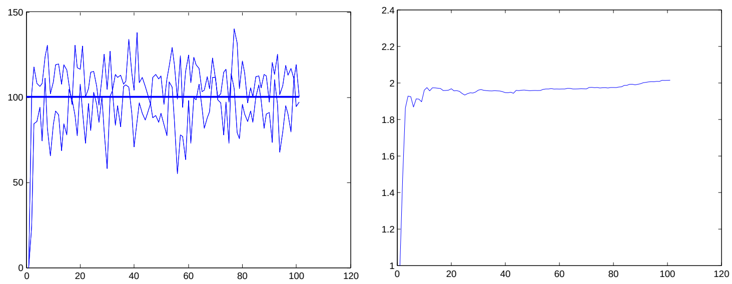
Alg. 2: *Kiefer-Wolfowitz Algorithm.*



(a) *Size of target population* N^* *(horizontal line); evolution of* $k \mapsto \hat{N}^{\lambda_k^n - c_k}$ *and* $k \mapsto \hat{N}^{\lambda_k^n + c_k}$.

(b) $k \mapsto \lambda_k^n$.

Fig. 6 : Optimization birth rate: slow convergence.



(a) *Size of target population* N^* *(horizontal line); evolution of* $k \mapsto \hat{N}^{\lambda_k^n - c_k}$ *and* $k \mapsto \hat{N}^{\lambda_k^n + c_k}$.

(b) $k \mapsto \lambda_k^n$.

Fig. 7 : Optimization birth rate: speed convergence.

V. Conclusion

We applied the classical results on ponctuel processes and Markov processes of pure jump model for forest dynamics in the simple form. This approach is individual-centered: each tree (individual) is explicitly represented within a population. The basic mechanisms (birth, dispersal, natural death, death by competition) are explicitly described. Including the level of competition is explained as a death rate of $\lambda^c(x, \nu)$ which describes the competition exerted by the individual x from the forest ν.

Although very simple, the resulting model presents very interesting features. It helps to account for different behaviors (low or high dispersion, low or high competition).

In a numerical simulation, we describe a Monte Carlo algorithm in continuous time and continuous space. We found that, despite its simplicity, this model can account for a large variety of aggregations.

To fitting the model to data, we are interested in a stochastic gradient method which allows to "solve" a setting to achieve a given target feature. Here we have optimized the birth rate λ^n to reach a target population size N^* given.

Finally, it would be interesting: (i) taking into account the growth of individuals, (ii) treat the case of two or a few species. Before getting down to the use of such a model, it would be interesting to continue testing optimization of parameters by stochastic gradient technique. It would be conceivable to use such models in landscape ecology research and making decision in forest management.

References

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